WKB APPROXIMATION

METHOD
UNIT IV

MSC 202

BY:
BANDANA JADAWN

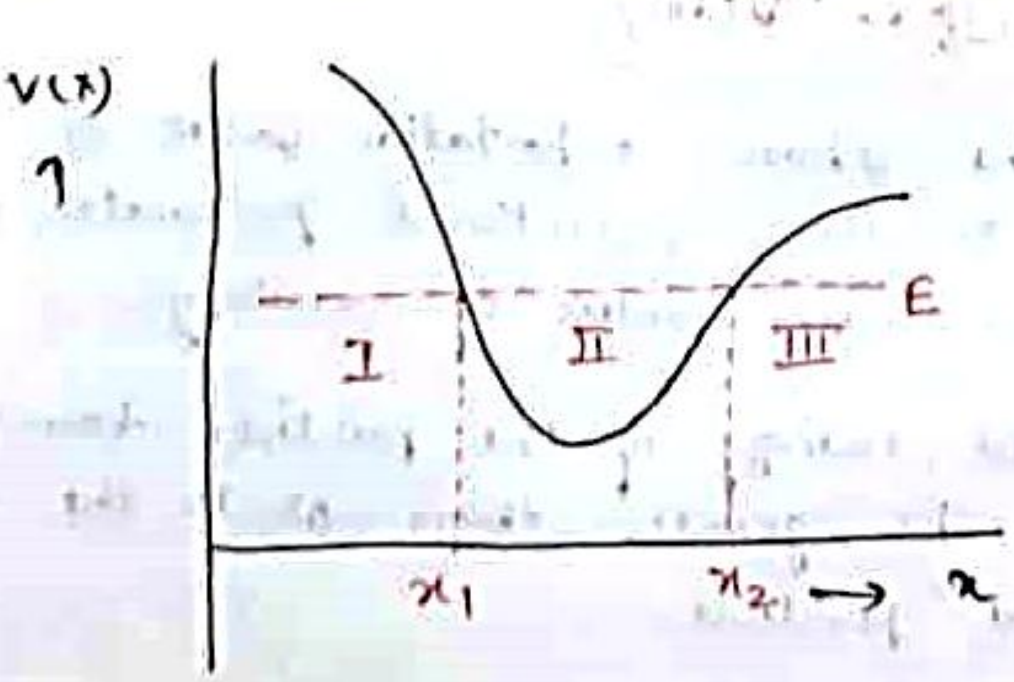
WKB METHOD [WENTZAL KRAMER BRILLOUINE]

WKB approximation method is also used to find the energy of a particle in various eigen states of a quantum mechanical system and to find the transmission coefficient for a potential barrier.
This method is applicable only for slowly varying potential and for slowly varying potential, the potential remains almost constant over a region of the order of de-broglie wavelength of the particle.
For $\lambda = 10^{-15}$ m, $\frac{dV}{dx}(10^{-15})$ should be almost constant.

The condition for slowly varying potential

$$\boxed{\lambda(x) \left|\frac{dV}{dx}\right| << |V(x)|}$$

For varying potentials λ is a function of x.
$V(x) \propto x$. For macroscopic objects λ is very small. The de-broglie wavelength of the object is very small compared to the dimension of the object. Hence, the potential under which particle is moving will remain constant over a region of the order of de-broglie wavelength. It is also called semi-classical approximation. (b/c it works in classical limit i.e. $\lambda \to 0$)



In region I : $E < V(x)$ and $-\infty < x < x_1$ This is classically forbidden region.

In region II : $E > V(x)$, $x_1 < x < x_2$ This is classically allowed region.

In region III : $E < V$
$x_2 < x < \infty$ This is classically forbidden region

According to classical theory particle will not exist in region I and region III because Kinetic Energy of particle will be negative in those regions. Therefore, the particle will be confined in region II. when particle reaches at points x_1 and x_2 it will reflected back. Hence, x_1 and x_2 are known as classical turning points of system. (At turning points $E = V$)

Validity :-

* It is valid in both classically allowed and classically forbidden region.

Imp

* $$\left|\frac{d\lambda}{dx}\right| << 1$$

or $$\left|\frac{1}{k^2}\cdot\frac{dk}{dx}\right| << 1$$

Here $\lambda = \frac{h}{p}$

$$\lambda = \frac{h}{\sqrt{2m(E-V(x))}}$$

* It is allowed where momentum ($p=0$) is zero b/c when $p=0$ then 'λ' tends to infinite. This will happen at classical turning points. Therefore, WKB approx. method is not valid at classical turning points.

Quantization condition of energy levels in W.K.B. approximation

$$\boxed{\frac{1}{\hbar}\int_{x_1}^{x_2} p(x')\,dx' + \beta_1 + \beta_2 = \begin{cases} (n+1)\pi & n = 0,1,2,\ldots \\ n\pi & n = 1,2,3,\ldots \end{cases}}$$

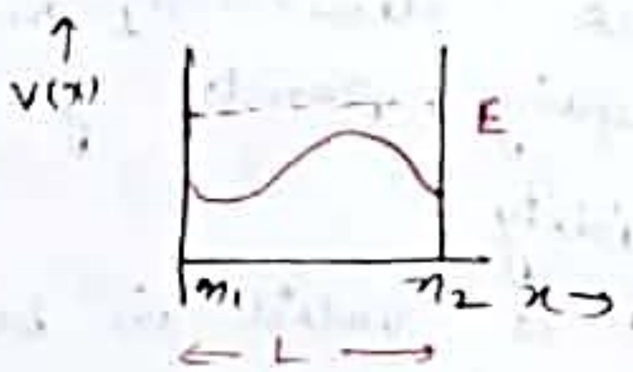
This is the quantization condition.
where β_1 is phase factor for first turning point and β_2 is phase factor for second turning point.

where, $\beta = \begin{cases} 0 & \text{rigid boundary} \quad V(x) = \text{infinite} \\ \pi/4 & \text{non-rigid boundary} \quad V(x) = \text{finite} \end{cases}$

Applications :

1. Bound state with potential wells with two rigid walls

Here are two rigid walls one at $x = x_1$ and other at $x = x_2$.



By using quantization condition

$$B_1 + \int p\,dx + B_2 = (n+1)\pi\,\hbar$$

$$p = \sqrt{2m(E-V)}$$

$$\int_{x_1}^{x_2} \sqrt{2m(E-V)}\,dx + 0 + 0 = \begin{cases} (n+1)\pi\hbar & n = 0,1,2,\ldots \\ n\pi\hbar & n = 1,2,3\ldots \end{cases}$$

$\because V = 0$

$$\int_0^L \sqrt{2mE}\,dx = \sqrt{2mE}\int_0^L dx$$

$$= \sqrt{2mE}\,.\,[L] = n\pi\hbar \qquad n = 1,2,3\ldots$$

$$\Rightarrow \quad L\cdot\sqrt{2mE} = n\pi\hbar$$

$$\Rightarrow \quad 2mE = \frac{n^2\pi^2\hbar^2}{L^2}$$

$$\Rightarrow \quad \boxed{E = \frac{n^2\pi^2\hbar^2}{2mL^2}}$$

This is the well known eigen energy value for 1-dim square well potential.

Note: short cut trick for solving WKB method

If $V(x) \propto x^m$

then dependency of Energy on the value of n-

$$\boxed{E_n \propto n^{2m/m+2}}$$


THANK
YOU
BY :BANDANA JADAWN